Section 5.2 *The Natural Logarithmic Function: Integration*

Log Rule for Integration

The differentiation rules

$$\frac{d}{dx}[\ln|x|] = \frac{1}{x} \qquad \text{and} \qquad \frac{d}{dx}[\ln|u|] = \frac{u'}{u}$$

that you studied in the preceding section produce the following integration rule.

THEOREM 5.5   Log Rule for Integration

Let u be a differentiable function of x.

1. $\displaystyle\int \frac{1}{x}\,dx = \ln|x| + C$ **2.** $\displaystyle\int \frac{1}{u}\,du = \ln|u| + C$

Ex.1 Using the Log Rule for Integration

Find $\displaystyle\int \frac{2}{x}\,dx =$

Ex.2 Using the Log Rule with a Change of Variables
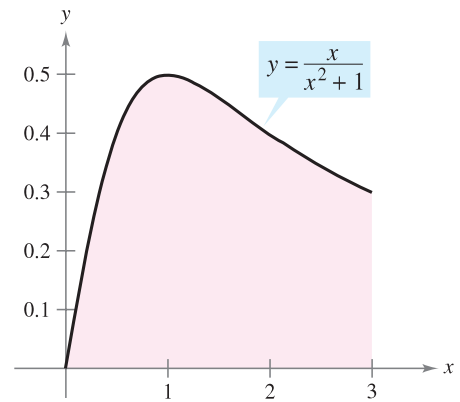
Find $\displaystyle\int \frac{1}{4x - 1}\,dx.$

Ex.3 Finding Area with the Log Rule

Find the area of the region bounded by the graph of
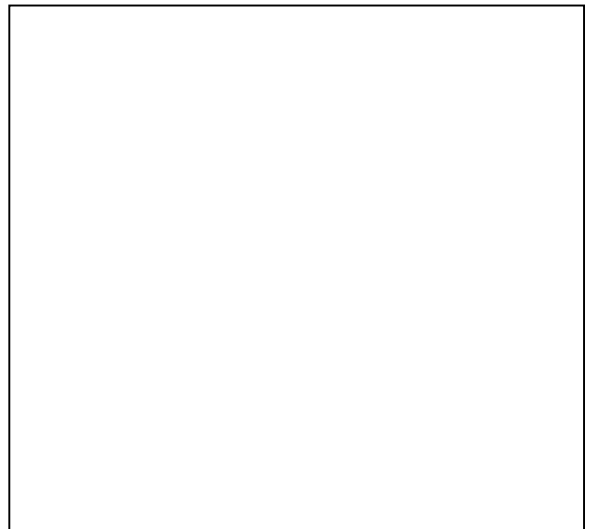
$$y = \frac{x}{x^2 + 1}$$

the x-axis, and the line $x = 3$.

$$\text{Area} = \int_0^3 \frac{x}{x^2 + 1} \, dx$$

The area of the region bounded by the graph of y, the x-axis, and $x = 3$ is $\frac{1}{2} \ln 10$.

Figure 5.8

Ex.4 Recognizing Quotient Forms of the Log Rule

a. $\displaystyle \int \frac{3x^2 + 1}{x^3 + x} \, dx =$

b. $\displaystyle \int \frac{\sec^2 x}{\tan x} \, dx =$

c. $\displaystyle \int \frac{x + 1}{x^2 + 2x} \, dx =$

d. $\displaystyle \int \frac{1}{3x + 2} \, dx =$

Integrals to which the Log Rule can be applied often appear in disguised form. For instance, if a rational function has a *numerator of degree greater than or equal to that of the denominator*, division may reveal a form to which you can apply the Log Rule. This is shown in Example 5.

Ex.5 Using Long Division Before Integration

Find $\displaystyle\int \frac{x^3 - 6x - 20}{x + 5}\, dx$

Ex.6 Change of Variables with the Log Rule

Find $\displaystyle\int \frac{2x}{(x+1)^2}\,dx.$

Ex.7 *u*-Substitution and the Log Rule

Solve the differential equation $\dfrac{dy}{dx} = \dfrac{1}{x \ln x}$.

Integrals of Trigonometric Functions

In Section 4.1, you looked at six trigonometric integration rules—the six that correspond directly to differentiation rules. With the Log Rule, you can now complete the set of basic trigonometric integration formulas.

Ex.8 Using a Trigonometric Identity

Find $\displaystyle\int \tan x \, dx$.

Ex.9 Derivation of the Secant Formula

Find $\int \sec x \, dx$.

Integrals of the Six Basic Trigonometric Functions

$\int \sin u \, du = -\cos u + C$ $\int \cos u \, du = \sin u + C$

$\int \tan u \, du = -\ln|\cos u| + C$ $\int \cot u \, du = \ln|\sin u| + C$

$\int \sec u \, du = \ln|\sec u + \tan u| + C$ $\int \csc u \, du = -\ln|\csc u + \cot u| + C$

Ex.10 Integrating Trigonometric Functions

Evaluate $\displaystyle\int_0^{\pi/4} \sqrt{1 + \tan^2 x}\, dx$.

Ex.11 Finding an Average Value

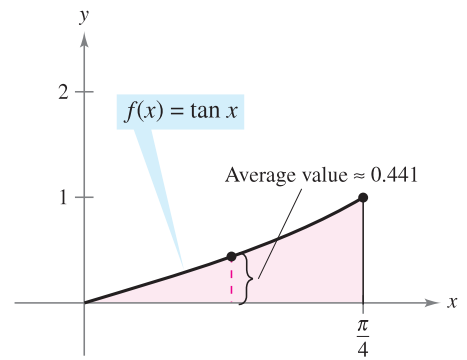Find the average value of $f(x) = \tan x$ on the interval $\left[0, \dfrac{\pi}{4}\right]$.

Figure 5.9